Eyeballing state dependence and unobserved heterogeneity in aggregate unemployment duration data

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EYEBALLING STATE DEPENDENCE AND UNOBSERVED HETEROGENEITY IN AGGREGATE UNEMPLOYMENT DURATION DATA

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I. INTRODUCTION

In the past decade, the analysis of unemployment durations has become widespread. One of the major issues in this literature concerns the distinction between state dependence of the hazard rate (i.e., dependence of the exit rate out of unemployment for a given individual on his elapsed duration of unemployment) and unobserved heterogeneity (for surveys, see for example Devine & Kiefer, 1991). Often, there is reason to believe that for a given individual the hazard rate decreases as a function of duration. For example, there may be stigma effects reducing the number of job opportunities for the long-term unemployed (see for example Vishwanath, 1989; Van den Berg, 1994). On the other hand, the presence of unobserved heterogeneity in the distribution of the duration variable causes the hazard rate of the distribution of observed durations to decrease as well. This follows from the fact that on average

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individuals with the largest hazard rate leave unemployment first. Obviously, from a policy point of view, it is important to know the relative importance of state dependence (also called genuine duration dependence) on the one hand, and unobserved heterogeneity on the other. For example, if state dependence is the dominant factor, then efforts may be concentrated on the long-term unemployed, while otherwise it may be useful to screen short-term unemployed and concentrate efforts on those with bad characteristics. Also, the degree of state dependence is of importance for macro analyses of the labor market (see, e.g., Layard, Nickell, & Jackman, 1991; Jackman & Savouri, 1992). However, since both factors affect the hazard rate in a similar way, it seems to be hard to distinguish empirically between them.

With a sufficient amount of data it is possible to address these issues by way of sophisticated empirical analyses using high-tech econometric methods. As is well known, state dependence and the distribution of unobserved heterogeneity are identified from typical micro duration data if a Mixed Proportional Hazard (MPH) model framework is adopted (see Lancaster, 1990 for a survey). Van den Berg and Van Ours (1996) provide a nonparametric estimation method that needs high-quality aggregate data on time series of exit rates from a number of unemployment duration classes. In practice, however, the availability of data may be limited, or the scope of the study at hand does not allow a detailed and sophisticated empirical analysis of duration models even though addressing state dependence and unobserved heterogeneity is warranted. In the present chapter we present easy-to-perform nonparametric eyeball checks that can be used to detect whether there is state dependence and/or unobserved heterogeneity and that have limited data requirements. In particular, these checks merely need time series of the average exit rate from unemployment and the exit rates from the first two (for example) quarterly duration classes. Such data may be derived from administrative files that are easily accessible for different countries and on different types of individuals.

So-called eyeball checks are based on easy-to-detect properties of data summarized in graphs and tables. By examining particular characteristics of graphs or tables of the aggregate data mentioned above, one can detect in a simple way whether there is evidence for state dependence and/or unobserved heterogeneity. Aggregate (or gross, or macro) data have the advantage that they provide the exact values of the exit probabilities for the different duration classes (averaged over unobserved heterogeneity), in the sense that the whole population of unemployed is incorporated in the data. It should be noted from the outset that eyeball checks are not tests in the usual statistical sense.

A marked advantage of the eyeball checks proposed in this paper is that they do not rely on parametric functional-form assumptions on the determinants of the exit rate (apart from the MPH assumption). In the literature up to date, all studies based on micro data rely on such assumptions for at least one determinant of the exit rate. For example, typical choices are: (*i*) Weibull duration dependence (*ii*) a discrete distribution of unobserved heterogeneity with a fixed number of points of support, or (*iii*) loglinear dependence of the exit rate on observed explanatory variables (see the surveys mentioned above and the references therein). Intuitively it is clear that the results on the degree of state dependence and unobserved heterogeneity may be extremely sensitive with respect to misspecification of the corresponding functional forms (see e.g., Ridder, 1987 for some evidence).

In this chapter we present an eyeball check for state dependence that can be traced back to a somewhat informally presented idea in Jackman and Layard (1991). We characterize the model framework within which the eyeball check is meaningful, and, by way of a formal analysis, we derive precise conditions under which this eyeball check is informative on the presence of state dependence. In addition, we examine to what extent this check can be used to infer the sign of the state dependence (i.e., whether the individual exit rate increases or decreases as a function of duration). We also present and analyze an eyeball check for unobserved heterogeneity. It turns out that in certain cases this check is also informative on the validity of the MPH assumption.

Section II explains the gist of the eyeball checks by way of some numerical examples. Section III presents the model framework. Basically, the model is a Mixed Proportional Hazard (MPH) model in which calendar time replaces the role of the observed explanatory (x) variables. This model has also been used in Van den Berg and Van Ours (1994, 1996). Those articles apply formal econometric tools to estimate a model using extensive aggregate data on time series of exit rates from a number of unemployment duration classes. As such, they are fully complementary to the present chapter.

In Section IV we present and analyze the eyeball checks. Section V contains empirical illustrations. We use data from France, the United Kingdom and The Netherlands. Section VI concludes.

II. THE GIST OF THE EYEBALL CHECKS

In this section we present the gist of our method by way of numerical examples. These examples quantify the whole unemployment duration model. We therefore start with an introductory subsection on notation and the model framework. Section IV contains the formal general analysis.

A. Some Notation

We use two measures of time. The variable t denotes the duration of unemployment, as measured from the moment the individual becomes unemployed. The variable τ denotes calendar time, which has its origin somewhere in the past. For simplicity we take t and τ to have the same measurement scale, apart from the difference in origin. Both t and τ are discrete variables. As an example, consider an individual who is unemployed for t periods at calendar time τ. If he fails to leave unemployment in period t, he will be unemployed for $t + 1$ periods at calendar time $\tau + 1$.

Ideally, aggregate data give the total numbers of individuals in the labor market who are unemployed for t periods of time ($t = 0, 1, 2, \ldots$) at calendar times τ, $\tau + 1$, $\tau + 2$, and so forth. By comparing the number of individuals who are unemployed for t periods of time at τ to the number unemployed for $t + 1$ periods at $\tau + 1$, one observes the exit probability out of unemployment at calendar time τ for duration t. In other words, one observes the conditional probability that an individual leaves unemployment when being unemployed for t periods, when calendar time equals τ at the moment of exit, for different values of t and τ.

We consider τ to be an explanatory variable for individual unemployment durations t, in the sense that the exit probability out of unemployment for individuals with duration t may vary over calendar time. Thus, calendar time is assumed to capture cyclical macro effects on individual exit probabilities out of unemployment as well as structural changes influencing these probabilities.

The model framework we adopt for individual unemployment durations expresses the distribution of this duration (or, equivalently, variation in the individual exit probabilities) in terms of observed and unobserved individual characteristics, calendar time, and the state dependence pattern. Usually, gross data do not contain information on individual characteristics that could be used as explanatory variables. At best, gross figures are collected separately for a few different strata of individuals. In the sequel we will therefore suppress the conditioning on the prevailing value of observed explanatory variables.

We denote the unobserved heterogeneity variable by v. Now consider an individual with unobserved characteristics v who is unemployed for t periods when calendar time equals τ. We denote the conditional probability that this individual leaves unemployment after t periods of unemployment by $\Theta(t|\tau,v)$. By definition, this is the exit probability out of unemployment (or hazard) at t conditional on τ and v. The unemployment duration density conditional on calendar time and conditional on v can be constructed from these exit probabilities. For example, the probability that unemployment duration equals t, when calendar time was $\tau - t$ at the moment of inflow into unemployment, conditional on v, equals

$$\theta(t|\tau,v) \cdot \prod_{i=0}^{t-1} (1 - \theta(i|\tau - t + i, v)) \qquad (2.1)$$

for all $t \in \{0, 1, \ldots\}$. We take the product term to be one if $t = 0$.

We assume that $\theta(t|\tau,v)$ has a mixed proportional hazard specification, that is, that $\theta(t|\tau,v)$ can be written as a product $\psi_1(t) \cdot \psi_2(\tau) \cdot v$. If ψ_1 is constant then there is no state dependence. If the distribution of v is degenerate then there is no unobserved heterogeneity. Section III contains a more formal discussion.

As mentioned above, the data provide observations on the conditional probabilities that individuals leave unemployment when being unemployed for t periods, when calendar time equals τ at the moment of exit, for different values of t and τ. These probabilities are unconditional on the unobserved heterogeneity term v, and will be denoted by $\theta(t|\tau)$. To express these observed exit probabilities $\theta(t|\tau)$ in terms of the exit probabilities $\theta(t|\tau,v)$, we have to integrate v out of the latter (see Section III).

B. Description of the Gist of the Eyeball Checks

In the numerical examples, we apply the eyeball checks in a few different quantified models. To do so we have to calculate values of the so-called eyeball check indicators in these models. These indicators are such that they can be calculated from observable data in empirical applications.

For each model we calculate the indicators at two different points of time. Specifically, we do as if we have "data" from two moments of time at which the labor market is in a different steady state. Somewhat loosely, we define a steady state as a period of time within which the inflow into unemployment is constant over time and equal to the outflow from unemployment, and within which the exit probabilities do not vary as a function of calendar time. The latter means that the value of the calendar time component of the exit probability $\psi_2(\tau)$ is constant within a steady state s. Because of this, exit probabilities $\theta(t|\tau,v)$ and $\theta(t|\tau)$ can be denoted by $\theta(t|s,v)$ and $\theta(t|s)$, and $\psi_2(\tau)$ can be denoted by a_s, when τ is in steady state s. The higher a_s, the more "favorable" the steady state. The "unfavorable" steady state is characterized by a low initial exit probability $\theta(0|s)$ (we take 0.30), and the "favorable" steady state is characterized by a high (0.60) initial exit probability.[1]

We use two eyeball-indicators. First, a state dependence indicator that is defined as $\theta(0|s)/\overline{\theta}(s)$, which is the ratio of the exit probability for the first duration class and the overall probability of exiting unemployment within one time period. Second, an unobserved heterogeneity indicator that is defined as $\theta(1|s)/\theta(0|s)$, which is the ratio of the exit probabilities for the second and first duration classes, respectively. The eyeball checks consist of comparing the values of the indicators for different steady states s. Notice that our eyeball indicators require only limited information with respect to the exit probabilities from unemployment. In particular, $\overline{\theta}(s)$ can readily be observed in aggregate data even if $\theta(t|s)$ is not observed for every t separately.

In the examples we consider three special cases: (*i*) no state dependence—no unobserved heterogeneity, (*ii*) no state dependence—unobserved heterogeneity, and (*iii*) state dependence—no unobserved heterogeneity. The first two columns of Table 1 give a numerical example of case (*i*). The exit probability is the same for every individual and does not change over the duration. As a result, both eyeball indicators are exactly equal to 1.

Table 1. Numerical Examples

	Case (i) No State Dependence, No Unobs. Heterog.		Case (ii) No State Dependence, Unobserved Herterog.					Case (iii) State Dependence, No Unobs. Heterog.	
	$t\mid s, v_i = t\mid s$		$t\mid s, v_1$		$t\mid s, v_2$		$t\mid s$	$t\mid s, v_i = t\mid s$	
st. state a_1	θ	U	θ	U	θ	U	θ	θ	U
$t = 0$	0.3	100	0.2	100	0.4	100	0.30	0.30	100
1	0.3	70	0.2	80	0.4	60	0.29	0.27	70
2	0.3	49	0.2	64	0.4	36	0.27	0.27	51
3	0.3	34	0.2	51	0.4	22	0.26	0.27	35
	$t\mid s, v_i = t\mid s$		$t\mid s, v_1$		$t\mid s, v_2$		$t\mid s$	$t\mid s, v_i = t\mid s$	
st. state a_2	q	U	θ	U	θ	U	θ	θ	U
$t = 0$	0.6	100	0.4	100	0.8	100	0.60	0.60	100
1	0.6	40	0.4	60	0.8	20	0.50	0.54	40
2	0.6	16	0.4	36	0.8	4	0.44	0.54	18
3	0.6	6	0.4	22	0.8	1	0.41	0.54	8

Notes: We normalize by taking $\psi_1(0) = 1$ and $E(v) = 1$, so $a_1 = 0.3$ and $a_2 = 0.6$. In case (ii), $v_1 = 2/3$ and $v_2 = 4/3$, and $\Pr(v = v_1) = \Pr(v = v_2) = 1/2$. In case (iii), $\psi_1(t) = 0.9$ for $t > 1$. $U(t\mid s, v_i)$ denotes the number of unemployed with $v = v_i$ in duration class t in steady state s, with $U(0\mid s, v_i)$ normalized. In cases (i) and (iii), $\theta(t\mid s, v_i)$ and $U(t\mid s, v_i)$ do not depend on v_i, and $\theta(t\mid s)$ equals $\theta(t\mid s, v_i)$.

Now, consider case (*ii*). We assume that unobserved heterogeneity v has a distribution with two points of support: v_1 and v_2. This implies that there are two homogeneous groups of unemployed workers. We assume that the exit probability of the second group of workers is twice as large as the exit probability of the first group of workers, so $v_2 = 2v_1$. We also assume that these groups are of equal size in the inflow into unemployment. As indicated in Table 2, the state dependence indicator is now equal to 1.12 in both states of the labor market (while it is not equal in both states in case (*iii*) of state dependence). From this we obtain our first conclusion: if there is no state dependence then the state dependence indicator does not depend on the state of the labor market. We will show below that in many cases the reverse is true as well. In sum, the state dependence indicator detects whether there is state dependence.

What is the intuition behind this state dependence eyeball check? Basically, it exploits implications of the MPH assumption. First of all, $\theta(0\mid s)$ is always proportional to a_s, because $\theta(0\mid s,v)$ equals $\psi_1(0) \cdot a_s \cdot v$ and there is no dynamic selection at $t = 0$ so that $\theta(0\mid s) = E_v(\theta(0\mid s,v))$. Secondly, if there is no state dependence then the expectation[2] of $t + 1$ conditional on s and v is proportional to $1/a_s$. Thus, the expectation of $t + 1$ conditional on just s is also proportional to $1/a_s$. In a steady state in a discrete time framework, the size of the stock of unemployed equals the size of the flow times the mean duration plus one. By dividing both sides of this equality by the size of the stock we obtain that the mean duration plus one is equal to one over the overall exit probability. Thus, the overall exit probability is proportional to a_s. So if there is no state dependence then both $\theta(0|s)$ and $\bar{\theta}(s)$ are proportional to a_s. If there is state dependence then the overall exit probability is generally not proportional to a_s, because then the expectation of $t + 1$ conditional on s and v is generally not proportional to $1/a_s$ anymore.

Now, consider case (*iii*). We assume stepwise state dependence where the exit probability declines with 10 percent when going from the first to the second duration class, to remain constant after that. From Table 2 it appears that in case (*iii*) the unobserved heterogeneity indicator is the same for both states of the labor market (while it is not the same for both states in case (*ii*) of unobserved heterogeneity). From this we obtain our second conclusion: if there is no unobserved heterogeneity then the unobserved heterogeneity indicator does not depend on the state of the labor market. We will show below that the reverse is true as well. In sum, the unobserved heterogeneity indicator detects whether there is unobserved heterogeneity.

What is the intuition behind this unobserved heterogeneity eyeball check? First of all, recall that $\theta(0|s)$ is always proportional to a_s. If there is no unobserved heterogeneity then obviously all $\theta(t|s)$ are proportional to a_s. If there *is* unobserved heterogeneity then, as noted in the introduction, this causes the observed exit probability $\theta(t|s)$ to decrease as a function of duration t, because of dynamic selection. However, in presence of unobserved heterogeneity, the aggregate exit probability falls less sharply when going from $t = 0$ to $t = 1$ in an "unfavorable" steady state (low a_s), than it does in a "favorable" steady state. This is because in an unfavorable steady state the initial weeding out of individuals with a high quality (i.e., a large v) cannot occur as fast as in the other case. In other words, the dynamic selection occurs at a higher speed in a favorable steady state.

Note that this suggests that the unobserved heterogeneity indicator is always higher in the less favorable steady state. This is confirmed by the results in Table 2. From this we obtain our third conclusion: in the Mixed Proportional Hazard framework, the unobserved heterogeneity indicator is not positively related to a_s. Below we consider an alternative model framework for which the latter is violated. In conclusion, the relation between the unobserved heterogeneity indicator and a_s can be informative on the validity of the MPH assumption.

Let us return to the state dependence indicator. Table 2 suggests that the state dependence indicator is smaller in the more favorable steady state. From this we obtain our fourth conclusion: if there is negative state dependence then there is a negative relation between the state dependence indicator and a_s. Below we return to this, and we show that if there is positive state dependence then in general there

Table 2. Numerical Examples Eyeball Checks

	Case (i) *No State Dependence, No Unobs. Heterog.*		*Case (ii)* *No State Dependence, Unobserved Herterog.*		Case (*iii*) *State Dependence, No Unobs. Heterog.*	
steady state	a_1	a_2	a_1	a_2	a_1	a_2
$\theta(0\|s)/\overline{\theta}(s)$	1	1	1.12	1.12	1.08	1.04
$\theta(1\|s)/\theta(0\|s)$	1	1	0.95	0.83	0.90	0.90

Note: see Table 1.

is a positive relation. In conclusion, the relation between the state dependence indicator and a_s is informative on the sign of the state dependence.

III. THE MODEL

In this section we formally present the unemployment duration model framework and we discuss the underlying assumptions.

Assumptions

1. MPH: $\theta(t|\tau,v)$ has a mixed proportional hazard specification, that is, there are functions ψ_1 and ψ_2 such that

$$\theta(t|\tau,v) = \psi_1(t) \cdot \psi_2(\tau) \cdot v \tag{3.1}$$

with ψ_1 and ψ_2 positive and uniformly bounded from above. Further, the distribution of v is such that, for every t and τ, $Pr(0 < \theta(t|\tau,v) < 1) = 1$.

2. Invariance of v: the individual value of v does not change during unemployment, and the distribution of v in the inflow into unemployment does not depend on the moment of inflow.

3. Variation over calendar time: ψ_2 is not constant.

Assumption 1 is reminiscent of the standard MPH assumption in reduced-form duration models for micro duration data (see Lancaster, 1990 for an extensive survey of such models). In models for micro duration data, dependence on calendar time is usually ignored, and the role of τ in the model above is replaced by the role of observed explanatory variables x. An important difference between the present model and MPH models for micro data is that here we have discrete time, whereas in micro studies time is usually treated as continuous. Because of this we have to introduce the last line of Assumption 1. Note that it implies that the support of v is bounded. This in turn implies that all moments of v exist.

The present model should be regarded as a flexible accounting device for discrete time data, with an appealing interpretation. Recall from Section II that one of the eyeball checks can be informative on the validity of the MPH model specification. We return to this below.

The second part of Assumption 2 states that the distribution of v in the inflow is the same regardless of the moment of inflow. So, it rules out that there are cohort effects in the distribution of the unobserved heterogeneity term. There is abundant evidence that the season at the moment of inflow influences the unemployment duration (see e.g., Van den Berg & Van Ours, 1994). A possible explanation for this is that the composition of the inflow, as far as unobserved characteristics are concerned, varies over the seasons. This would violate Assumption 2. We ignore this issue in the theoretical analysis in this paper, while in the empirical illustration we seasonally adjust the data.

Assumption 3 is similar to the identifying assumption in the micro continuous-time MPH model that there is dispersion of observed explanatory variables. Note that sufficient for Assumption 3 is that there is a data point τ_0 somewhere in the time series such that $\psi_2(\tau) = a_1$ for $\tau < \tau_0$ and $\psi_2(\tau) = a_2$ for $\tau \geq \tau_0$.

To express the observed exit probabilities $\theta(t|\tau)$ in terms of the exit probabilities $\theta(t|\tau,v)$, we have to integrate v out of the latter. Let $\boldsymbol{t}$ denote the random unemployment duration, and t its realization. In obvious notation, there holds that

$$\theta(t|\tau) \equiv \frac{\Pr(\boldsymbol{t} = t|\text{inflow at } \tau - t)}{\Pr(\boldsymbol{t} \geq t|\text{inflow at } \tau - t)} \equiv \frac{E_v(\Pr(\boldsymbol{t} = t|\text{inflow at } \tau - t;v))}{E_v(\Pr(\boldsymbol{t} = t|\text{inflow at } \tau - t;v))} \tag{3.2}$$

in which the expectations $E_v(\cdot)$ are taken with respect to the distribution of v in the inflow into unemployment. Both $\Pr(\boldsymbol{t} = t|\text{inflow at } \tau - t;v)$ and $\Pr(\boldsymbol{t} \geq t \mid \text{inflow at } \tau - t;\, v)$ can be expressed in terms of $\theta(t|\tau,v)$ (note that equation (2.1) gives $\Pr(\boldsymbol{t} = t|\text{inflow at } \tau - t;v)$). By doing this, and by substituting equation (3.1), we get

$$\theta(t|\tau) = \frac{\psi_1(t) \cdot \psi_2(\tau) \cdot E_v\left[v \cdot \prod_{i=0}^{t-1} [1 - \psi_1(i) \cdot \psi_2(\tau - t + i) \cdot v] \right]}{E_v\left[\prod_{i=0}^{t-1} [1 - \psi_1(i) \cdot \psi_2(\tau - t + i) \cdot v] \right]} \tag{3.3}$$

To avoid confusion, note that even though t enters the argument of ψ_2 in (3.3), ψ_2 is a function of calendar time only. From (3.3) it follows that $\theta(t|\tau)$ can be expressed in terms of the "structural functions" ψ_1, ψ_2 and the distribution function $G(v)$ of v. We denote $E_v(v^i)$ by μ_i.

IV. THE EYEBALL CHECKS

A. The Eyeball Check for State Dependence

In this subsection we present an eyeball check for state dependence that can be traced back to an idea in Jackman and Layard (1991).[3] The exposition below is more formal than in their study and highlights the assumptions needed. In particular, for the eyeball check to be sensible we have to strengthen Assumption 3 and add another assumption,

> 3′. Steady states: there are at least two calendar time intervals S_i such that $\psi_2(\tau) = a_{Si}$ for $\tau \in S_i$ and $a_{Si} \neq a_{Sj}$ for $i \neq j$. S_i is so large that it contains points such that all individuals who are unemployed at such a point have entered unemployment during S_i and will exit unemployment during S_i. For the cohorts entering and leaving unemployment in S_i the mean unemployment duration is finite.
>
> 4. Constant inflow: the inflow rate into unemployment for individuals with characteristics v does not depend on calendar time except possibly for the value of $\psi_2(\tau)$.

Thus, we have (at least) two steady states. If S_1 and S_2 are sufficiently large and if Assumptions 1 and 2 are valid then it is not difficult to check whether the first part of Assumption 3′ is true. It implies that for each cohort entering unemployment just after the beginning of S_i the observed unemployment distribution must be the same, while the distribution for cohorts entering just after the beginning of S_1 must differ from the distribution for cohorts entering just after the beginning of S_2.

Assumption 4 is similar to the standard "constant inflow rate" assumption made in empirical analyses of duration data from stock samples (see e.g., Heckman and Singer, 1984). Note that it refers to the size of the inflow, whereas Assumption 2 refers to the composition of the inflow. Assumption 4 implies that within a steady state the inflow rate is constant. So, if data on the inflow size are available then this assumption can be checked as well. It should be noted that the results below are not sensitive to violations of Assumption 4 provided that the periods of time in which the inflow rate is not constant are sufficiently far in the past.

As in Section II, $\theta(t|\tau,v)$, $\theta(t|\tau)$ and $\psi_2(\tau)$ will be denoted by $\theta(t|s,v)$, $\theta(t|s)$ and a_s, respectively, when τ is in steady state s. From (3.3), the observed exit probability for the newly unemployed equals

$$\theta(0|s) = \psi_1(0) \cdot a_s \cdot \mu_1 \tag{4.1}$$

We now derive an expression for the overall exit probability $\overline{\theta}(s)$ at a certain point of time τ that satisfies the second part of Assumption 3′. The overall exit probability at a certain point of time equals the proportion of individuals who are unemployed at that point who leave unemployment within one time period. Let p denote the elapsed unemployment duration of individuals in the stock of unemployed at a point of time. The overall exit probability equals the sum over all values of p of the probability that the elapsed duration of unemployment $\boldsymbol{p}$ equals p and the residual duration $\boldsymbol{r}$ equals zero, conditional on presence in the stock of unemployed at that particular point of time. From results in for example Heckman and Singer (1984) it can be inferred that under Assumptions 1, 2, 3′ and 4 this equals

$$\bar{\theta}(s) = \sum_{p=0}^{\infty} \Pr(\boldsymbol{p} = p, \boldsymbol{r} = 0|s) = \frac{\sum_{p=0}^{\infty} E_v\left[\psi_1(p) \cdot a_s \cdot v \cdot \prod_{i=0}^{p-1} [1 - \psi_1(i) \cdot a_s \cdot v]\right]}{\sum_{p=0}^{\infty} E_v\left[\prod_{i=0}^{p-1} [1 - \psi_1(i) \cdot a_s \cdot v]\right]} \tag{4.2}$$

in which the product terms are one if $p = 0$. By comparing equations (3.2), (3.3) and (4.2) it follows that the numerator in the r.h.s. of (4.2) equals one. Let $\boldsymbol{t}$ be the random variable denoting the duration of unemployment of individuals sampled in the inflow. The denominator in the r.h.s. of (4.2) equals the sum over all $t \geq 0$ of $\Pr(\boldsymbol{t} \geq t|s)$. It is easy to show that for a discrete nonnegative random variable $\boldsymbol{t}$ there holds that

$$\sum_{t=1}^{\infty} \Pr(\boldsymbol{t} \geq t) = \sum_{t=0}^{\infty} t \cdot \Pr(\boldsymbol{t} = t) \equiv E(\boldsymbol{t})$$

Thus,

$$\bar{\theta}(s) = 1/(E(\boldsymbol{t}|s) + 1) \tag{4.3}$$

which is positive by virtue of Assumption 3′.

Proposition 1. Let Assumptions 1, 2, 3′ and 4 be satisfied. If there is no state dependence then $\theta(0|s)/\bar{\theta}(s)$ does not depend on s. In that case this ratio is larger than or equal to one.

Proof. If there is no state dependence then $\boldsymbol{t}|s,v$ has a geometric distribution with parameter $\psi_1(0) \cdot a_s \cdot v$. Consequently, $E(\boldsymbol{t}|s) = E_v(E(\boldsymbol{t}|a_s,v)) = E_v((1 - \psi_1(0) \cdot a_s \cdot v)/(\psi_1(0) \cdot a_s \cdot v))$, so $\bar{\theta}(s) = \psi_1(0) \cdot a_s/E_v(1/v)$. (Note that $E_v(1/v) < \infty$ by virtue of Assumption 3′.) Combining this with (4.1) gives

$$\frac{\theta(0|s)}{\bar{\theta}(s)} = E_v(v) \cdot E_v(1/v)$$

which does not depend on s and which by virtue of Jensen's inequality is larger than or equal to one. □

From the result above it follows that if $\theta(0|s)/\overline{\theta}(s)$ does depend on s (i.e., varies between steady states) then there must be state dependence. It is clear that the attractiveness of a check based on this depends on the extent to which the reverse holds as well. We now examine this in some more detail. In the general model, the ratio $\theta(0|s)/\overline{\theta}(s)$ does not depend on s if and only if $a_s(E(t|s) + 1)$ does not depend on a_s. (Recall that $E(t|s) + 1$ equals the denominator of the r.h.s. of (4.2).) The derivative of $a_s(E(t|s) + 1)$ w.r.t. a_s equals

$$E(t+1|s) + a_s \cdot E_v\left[\sum_{t=1}^{\infty}\sum_{k=0}^{t-1} -\psi_1(k)\cdot v \cdot \prod_{\substack{i=0\\ i\neq k}}^{t-1}[1-\psi_1(i)\cdot a_s \cdot v]\right]$$

which can be rewritten as

$$E_v\sum_{t=0}^{\infty}\left\{\left[(t+1)\cdot\psi_1(t)\cdot a_s\cdot v - \sum_{k=0}^{t-1}\frac{\psi_1(k)\cdot a_s\cdot v}{1-\psi_1(k)\cdot a_s\cdot v}\right]\prod_{i=0}^{t-1}[1-\psi_1(i)\cdot a_s\cdot v]\right\} \tag{4.4}$$

in which the second summation is zero if $t = 0$. Apparently, it is not possible to formulate a transparent condition on the model primitives such that the expression above is nonzero. Simulations with the model suggest that if the state dependence is positive (i.e., if $\psi_1(t)$ is monotonically increasing in t, or, in other words, if the exit probability out of unemployment for a given individual increases as a function of unemployment duration) then $\theta(0|s)/\overline{\theta}(s)$ is strictly increasing in a_s, and if the state dependence is negative then the opposite holds.

In the limiting continuous-time model it is possible to formulate a simple condition on the state dependence function $\psi_1(t)$ such that $\theta(0|s)/\overline{\theta}(s)$ is strictly increasing (or strictly decreasing) in a_s for every a_s and for every possible distribution of v. (See Appendix 1 for details.) Somewhat informally, this condition states that if $\psi_1(t)$ is increasing at most or all points of time and never strongly decreasing, then $\theta(0|s)/\overline{\theta}(s)$ is strictly increasing in a_s. Also, if $\psi_1(t)$ is decreasing at most or all points of time and never strongly increasing, then $\theta(0|s)/\overline{\theta}(s)$ is strictly decreasing in a_s.

This suggests the following practical guidelines. First, if it can be reasonably assumed *a priori* that state dependence is monotone, then the eyeball check is informative on the sign of the state dependence. For example, if $\theta(0|s)/\overline{\theta}(s)$ for the steady state with high $\theta(0|s)$ is smaller than $\theta(0|s)/\overline{\theta}(s)$ for the steady state with low $\theta(0|s)$, then there is negative state dependence. (Instead of using $\theta(0|s)$ one may also use $\overline{\theta}(s)$ in this relationship, since both are increasing in the value of a_s (for $\overline{\theta}(s)$ this follows from equations (4.2) and (4.3)).) Secondly, if it is known that state dependence is virtually monotone, then comparing two steady states suffices to detect whether there is any state dependence, because then for any $s_1 \neq s_2$ there holds that $\theta(0|s_1)/\overline{\theta}(s_1) \neq \theta(0|s_2)/\overline{\theta}(s_2)$. So, the eyeball check discussed in this subsection has very high power against monotone state dependence. Thirdly, if one does not want to rule out that $\psi_1(t)$ is wildly non-monotone as a function of t, then one should use as many steady states s as possible when examining the behavior of $\theta(0|s)/\bar{\theta}(s)$.

B. The Eyeball Check for Unobserved Heterogeneity

For the eyeball check of unobserved heterogeneity there is no gain in adopting Assumptions 3′ or 4. The check is based on examining the cross effect of t and calendar time τ on the observed log exit probability log $\theta(t|\tau)$. For practical reasons it turns out to be more convenient to describe the check as a comparison of values of ratios $\theta(t|\tau)/\theta(t-1|\tau)$ for one calendar time point to the values for other calendar time points.

It is clear that, for a check based on cross effects to be sensible, $\theta(t|\tau)$ has to vary over calendar time. This is guaranteed by Assumption 3. Note that it is straightforward to verify whether $\psi_2(\tau_1) \neq \psi_2(\tau_2)$ for $\tau_1 \neq \tau_2$, since equation (3.3) implies that $\psi_2(\tau_1)/\psi_2(\tau_2) = \theta(0|\tau_1)/\theta(0|\tau_2)$, and the latter ratio can be observed.

Proposition 2. Let Assumptions 1, 2 and 3 be satisfied. If there is no unobserved heterogeneity then $\theta(t|\tau)/\theta(t-1|\tau)$ does not depend on τ for any $t \in \{1, 2, \ldots\}$. Further, if there is unobserved heterogeneity then $\theta(1|\tau)/\theta(0|\tau)$ depends on τ in the sense that whenever $\psi_2(\tau_1 - 1) \neq \psi_2(\tau_2 - 1)$ (which is true for at least some τ_1 and τ_2) then $\theta(1|\tau_1)/\theta(0|\tau_1) \neq \theta(1|\tau_2)/\theta(0|\tau_2)$.

Proof. If there is no unobserved heterogeneity then $\Pr(v = \mu_1) = 1$ and $\theta(t|\tau) = \theta(t|\tau,v) = \psi_1(t) \cdot \psi_2(\tau) \cdot \mu_1$. Consequently, $\theta(t|\tau)/\theta(t-1|\tau) = \psi_1(t)/\psi_1(t-1)$ which does not depend on τ. It remains to prove that if there is unobserved heterogeneity then $\theta(1|\tau)/\theta(0|\tau)$ does depend on τ in the sense described above. From equation (3.3) it follows that

$$\frac{\theta(1|\tau)}{\theta(0|\tau)} = \frac{\psi_1(1)}{\psi_1(0)} \cdot \frac{1 - \psi_1(0) \cdot \psi_2(\tau - 1) \cdot \mu_2/\mu_1}{1 - \psi_1(0) \cdot \psi_2(\tau - 1) \cdot \mu_1} \tag{4.5}$$

The derivative of the r.h.s. of (4.5) w.r.t. $\psi_2(\tau - 1)$ is proportional to $\mu_1^2 - \mu_2$. There is unobserved heterogeneity if and only if $\mathrm{Var}(v) > 0$, i.e., if and only if $\mu_2 > \mu_1^2$. So, $\theta(1|\tau)/\theta(0|\tau)$ is a non-constant monotone decreasing function of $\psi_2(\tau - 1)$ if and only if there is unobserved heterogeneity. Recall that by Assumption 3, $\psi_2(\tau - 1)$ varies over τ. (As we have seen, it is easy to detect τ_1 and τ_2 for which $\psi_2(\tau_1) \neq \psi_2(\tau_2)$.) □

From Proposition 2 it follows that if there is a $t \in \{1, 2, \ldots\}$ such that $\theta(t|\tau)/\theta(t-1|\tau)$ does depend on τ (i.e., varies over τ) then there must be unobserved heterogeneity. Further, for $t = 1$ the reverse is also true, that is, if $\theta(1|\tau)/\theta(0|\tau)$ does not depend on τ then there is no unobserved heterogeneity. In Appendix 2 we examine to what extent the reverse is also true for $t > 1$.

Basically, the presence of unobserved heterogeneity is identified from cross effects of t and τ in $\log \theta(t|\tau)$. State dependence and calendar time dependence are identified from the separate additive effects of t and τ in $\log \theta(t|\tau)$ (or, in other words, multiplicative effects in $\theta(t|\tau)$). From this, and from the discussion in Section II on the rationale behind the state dependence eyeball check, it is clear that the MPH assumption is crucial for identification of the structural functions. The proof of Proposition 2 provides a check on this assumption. From this proof it follows that $\theta(1|\tau)/\theta(0|\tau)$ cannot be increasing as a function of $\psi_2(\tau - 1)$. Thus, if the calendar time effect causes the exit probabilities to be small (e.g., in times of recession) then the aggregate exit probability falls less sharply when going from $t = 0$ to $t = 1$ than if the opposite case holds. This is because in a recession the initial weeding out of individuals with a high quality (i.e., a large v) cannot occur as fast as in the other case. Now suppose one observes that, for some τ_1 and τ_2, $\theta(0|\tau_1 - 1) > \theta(0|\tau_2 - 1)$. This is equivalent to $\psi_2(\tau_1 - 1) > \psi_2(\tau_2 - 1)$. If it is also observed that $\theta(1|\tau_1)/\theta(0|\tau_1) > \theta(1|\tau_2)/\theta(0|\tau_2)$ then that is evidence that the MPH assumption is violated.

The (economic-theoretical) ranking model of unemployment as developed by Blanchard and Diamond (1994) predicts that the aggregate $\theta(t|\tau)$ as a function of t is more decreasing in a recession than in a boom. (The intuition behind this is that, in a recession, applications by long term unemployed individuals will be more frequently turned down for the reason that a short term unemployed individual has applied as well.) This implication of the ranking model is opposite to the implication of the MPH model for $\theta(t|\tau)$ as a function of t, and is regarded to be the main testable implication of the ranking model (see also Layard, Nickell, & Jackman, 1991). The ranking model does not assume unobserved individual-specific characteristics. In sum, the MPH model can be tested against this basic ranking model by examining whether $\theta(1|\tau)/\theta(0|\tau)$ decreases as a function of $\psi_2(\tau - 1)$ or not.

V. EMPIRICAL ILLUSTRATION

A. The Data

In Section II we illustrated the use of the eyeball checks by way of some numerical examples. In this section we apply the eyeball checks to data from France, the United Kingdom and The Netherlands. The primary aim is here to illustrate the use of these checks in an empirical setting, rather than to give detailed analyses of the labor markets in these countries.

For the eyeball checks two indicators are important, a state dependence and an unobserved heterogeneity indicator. The state dependence indicator $\theta(0|s)/\overline{\theta}(s)$ from Subsection IV.A refers to steady state periods s. Below we discuss the characteristics of these periods and how to find them empirically. For the moment we define the *state dependence indicator* for every calendar time point τ as the ratio of the exit probability for the first duration class and the overall exit probability, both at time τ: $\theta(0|\tau)/\overline{\theta}(\tau)$. The term $\overline{\theta}(\tau)$ can be expressed in a way similar to $\overline{\theta}(s)$ in equation (4.2). The *unobserved heterogeneity indicator* is the ratio $\theta(1|\tau)/\theta(0|\tau)$ of the exit probabilities for the second and first duration class, both at time τ.

To investigate the behavior of both indicators, we need time series information on aggregate numbers of unemployed individuals. The time frequency with which these numbers are observed has to be the same as the size of the duration classes. Let $U(t|\tau)$ denote the number of unemployed in duration class t at time τ. The exit probability for individuals in duration class t at time τ equals

$$\theta(t|\tau) = [U(t|\tau) - U(t+1|\tau+1)]/U(t|\tau)$$

For our analyses we use quarterly data. We have information on the number of unemployed in the first three quarterly duration classes and on the total number of unemployed at time τ ($U(\tau)$). We measure the overall exit probability from unemployment at time τ as:

$$\overline{\theta}(\tau) = [U(\tau) - U(\tau+1) + U(0|\tau+1)]/U(\tau)$$

So, we implicitly assume that the inflow into unemployment at time $\tau + 1$ is equal to the number of unemployed in the first duration class at time $\tau + 1$. In a discrete-time framework, this seems to be the most straightforward way to estimate the inflow size. See Jackman and Layard (1991) for an alternative approach. To eliminate seasonal fluctuations we use four-quarterly moving averages of exit probabilities.

Ideally, aggregate data provide exact values of exit probabilities. In reality, observations may differ from the values as predicted from the model because of measurement and specification errors. We assume somewhat loosely that the latter kind of errors do not dominate and that they are unsystematic. An advantage of eyeball checks is that they can be used in situations in which it is not aimed to construct and estimate formal models for these errors, or in situations in which informal data analyses are desired prior to more formal analyses.

We use data from three countries: France, the United Kingdom and The Netherlands. (For a number of other European countries it turned out to be impossible to obtain unemployment data meeting our requirements.) For each country we distinguish between male and female unemployed workers. The calendar time periods for which we have information cover a large part of the 1980s and the beginning of the 1990s (see Table 3). The French and Dutch data are collected by public employment offices and refer to registered unemployment. The United Kingdom data refer to benefit claimants.

Table 3 also lists averages of the overall quarterly exit probability as well as the exit probabilities for the first and second quarterly duration class. Comparing the

Table 3. Data Periods and Statistics

		Exit Probabilities (%)					
		Males			Females		
Country	*Data Period*	$\bar{\theta}(\tau)$	$\theta(0\mid\tau)$	$\theta(1\mid\tau)$	$\bar{\theta}(\tau)$	$\theta(0\mid\tau)$	$\theta(1\mid\tau)$
France	1983.4–92.1	31	38	34	26	32	30
United Kingdom	1984.3–92.2	27	42	32	34	46	34
Netherlands	1982.1–91.4	20	32	32	20	26	28

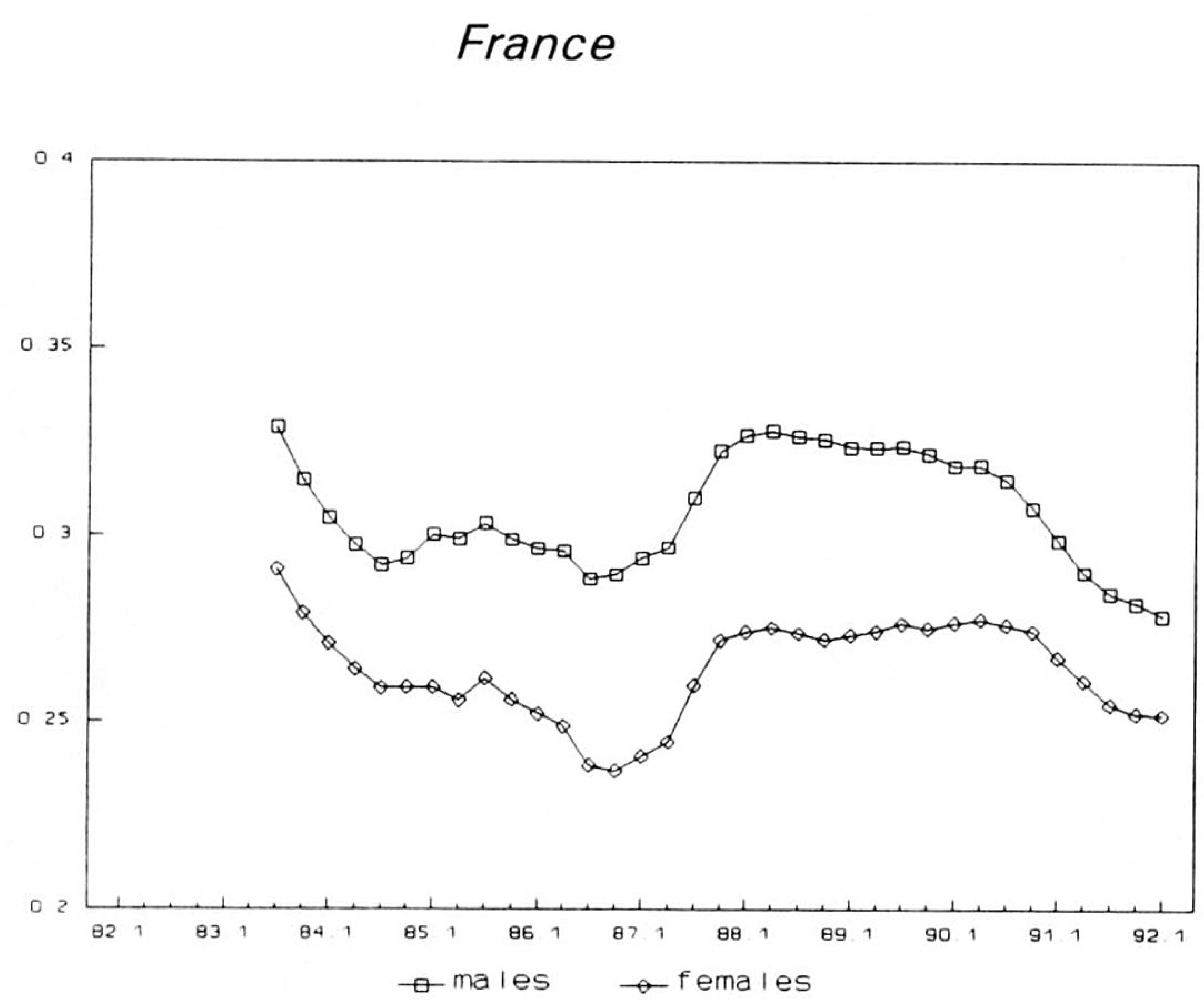
overall exit probability with the exit probabilities for the first two duration classes is informative about the way the exit probability changes over the duration of unemployment. For example, if the overall exit probability is smaller than those for the first two duration classes, then the exit probability declines after the first two quarters of unemployment.

The state dependence eyeball check compares different steady states. So, we have to find (at least) two steady state periods. (Alternatively, the data can be interpreted as a sequence of many steady states; see, e.g., Jackman & Layard, 1991 for such an approach.) In a steady state of unemployment, the stock, inflow and outflow are constant over a period of time. This means that we have to find two periods of time during which at least the overall exit probability out of unemployment is more or less constant. The length of the period of time during which stability is required depends on the average unemployment duration. If the average unemployment duration is short, the length of the required stability period is short too.

Taking the inverse of the overall exit probability as an indicator for the average duration of unemployment, we find substantial differences between countries. The average duration of unemployment in The Netherlands is five quarters and in France and the United Kingdom three quarters. Figure 1 shows the development of the overall exit probabilities out of unemployment. Note that the range of values on the horizontal axis and the length of the range of values on the vertical axis are the same in each graph. On the basis of these graphs we define the steady state periods indicated in Table 3. In general, the length of the steady state periods is about two to four times as long as the average unemployment duration, which seems reasonable. Note that the steady state periods lie within the long recession of the mid-1980s and the long boom in the late-1980s. It should also be noted that the results are insensitive with respect to the exact location of the steady state periods.

B. Eyeball Checks on State Dependence

The eyeball check for state dependence compares the values of the state dependence indicators for different steady states. Figure 2 shows the development of the state dependence indicator for each country and gender. Here, as well as in Figure 3, we use the rule that the length of the range of values on the vertical axis is the same for each of the six groups, and is equal to the largest of the six observed ranges.



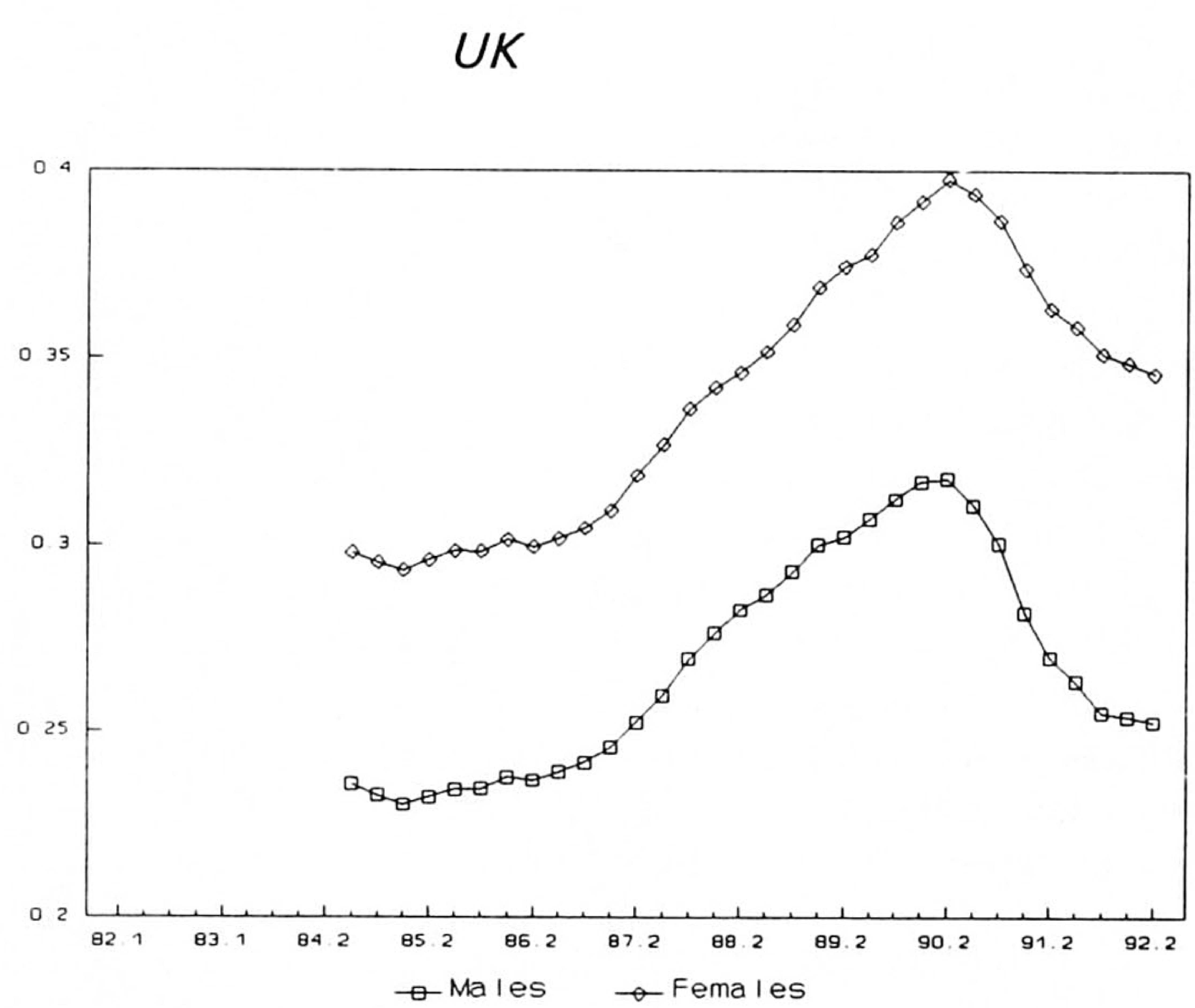


Figure 1. Overall quarterly exit probabilities out of unemployment; three countries.

The Netherlands

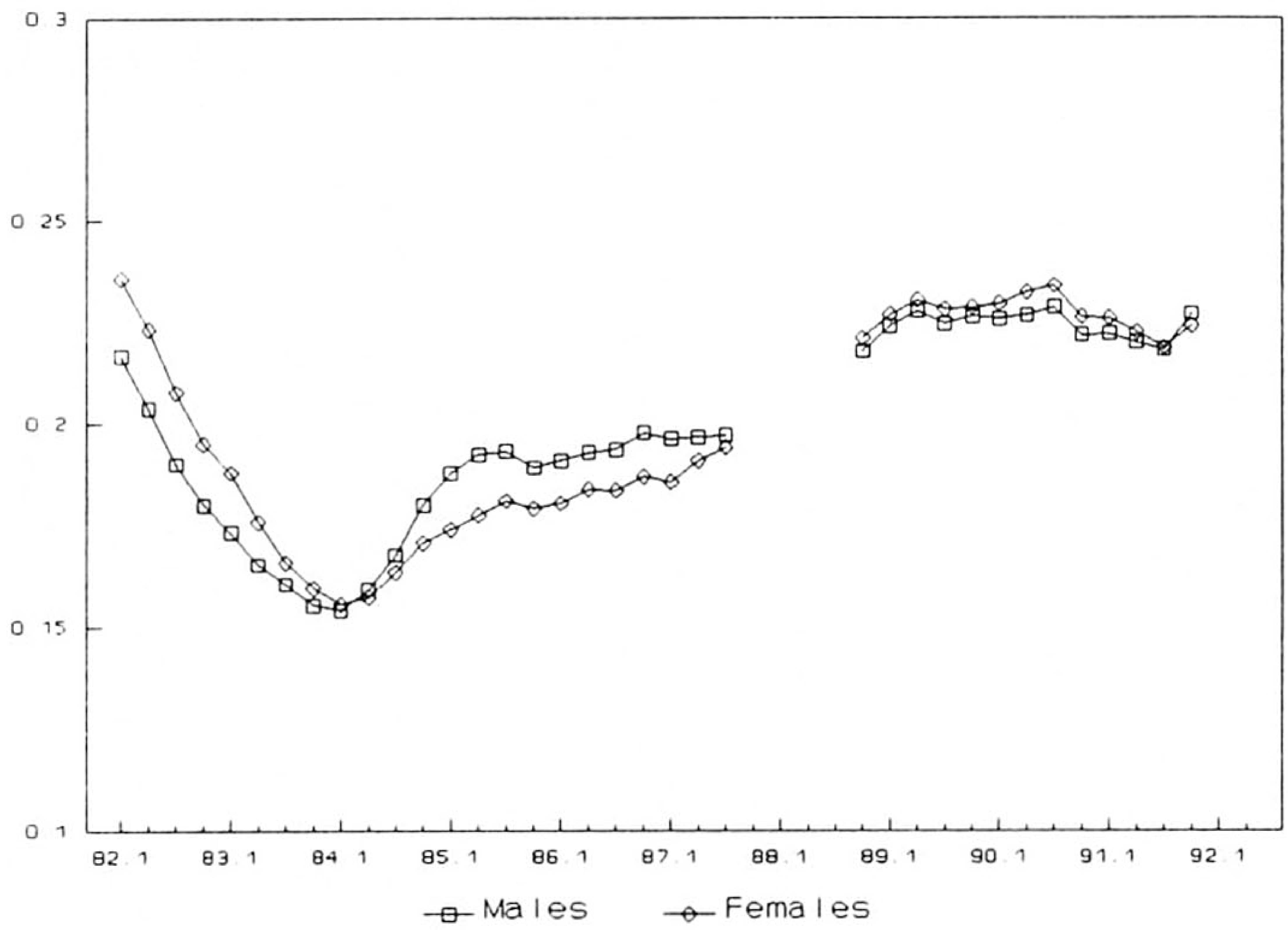


Figure 1. (Continued)

France

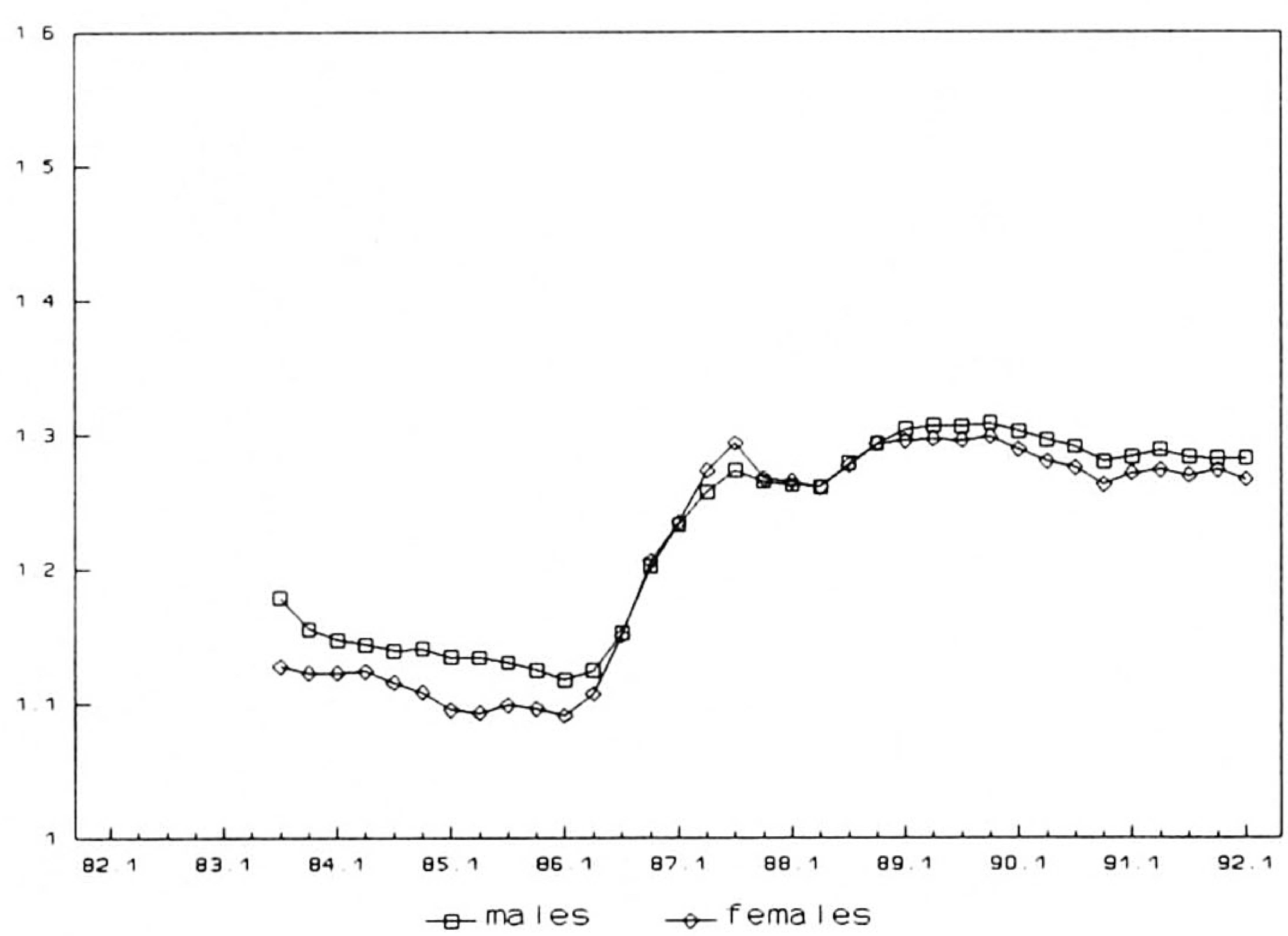
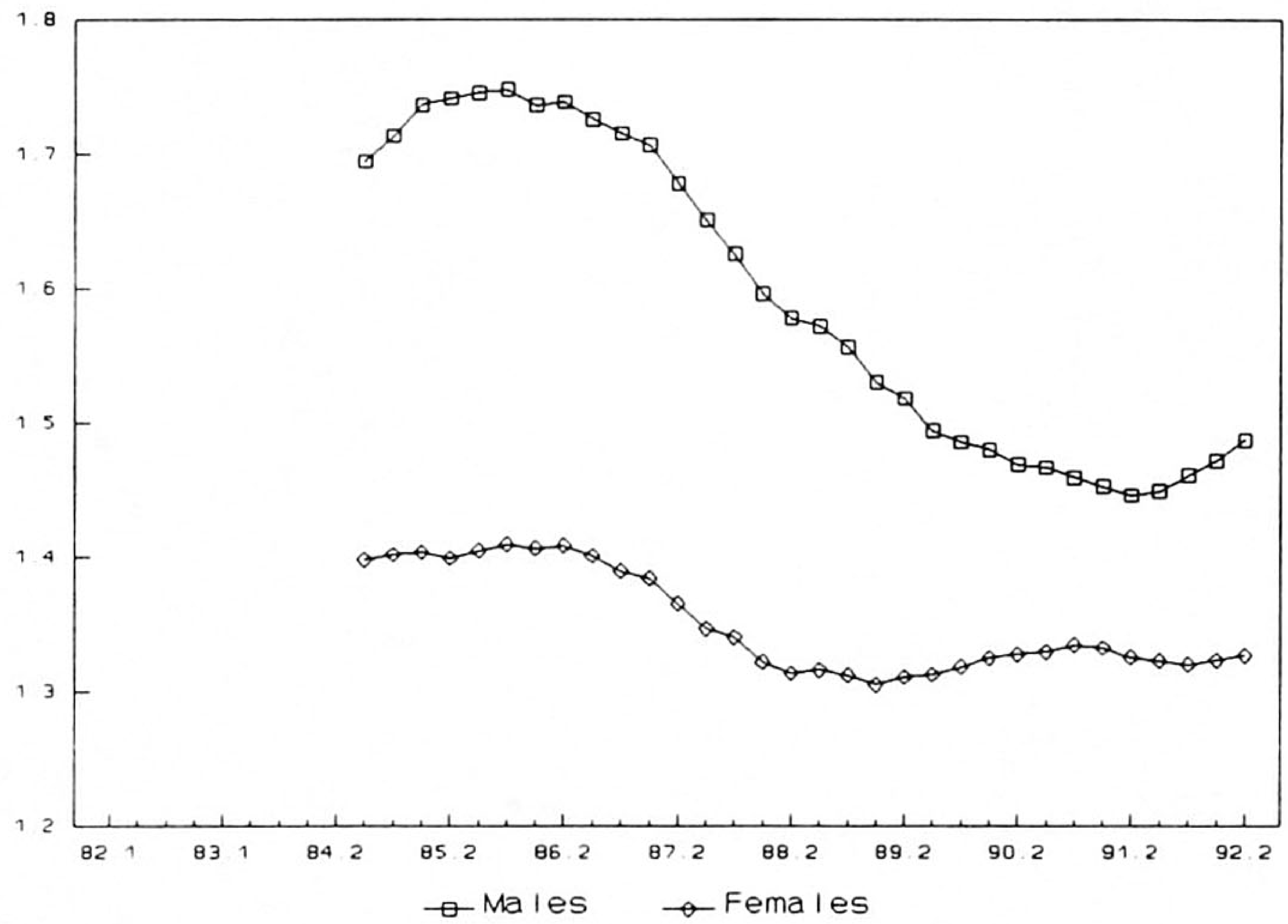


Figure 2. Indicators state dependence; three countries.

UK



The Netherlands

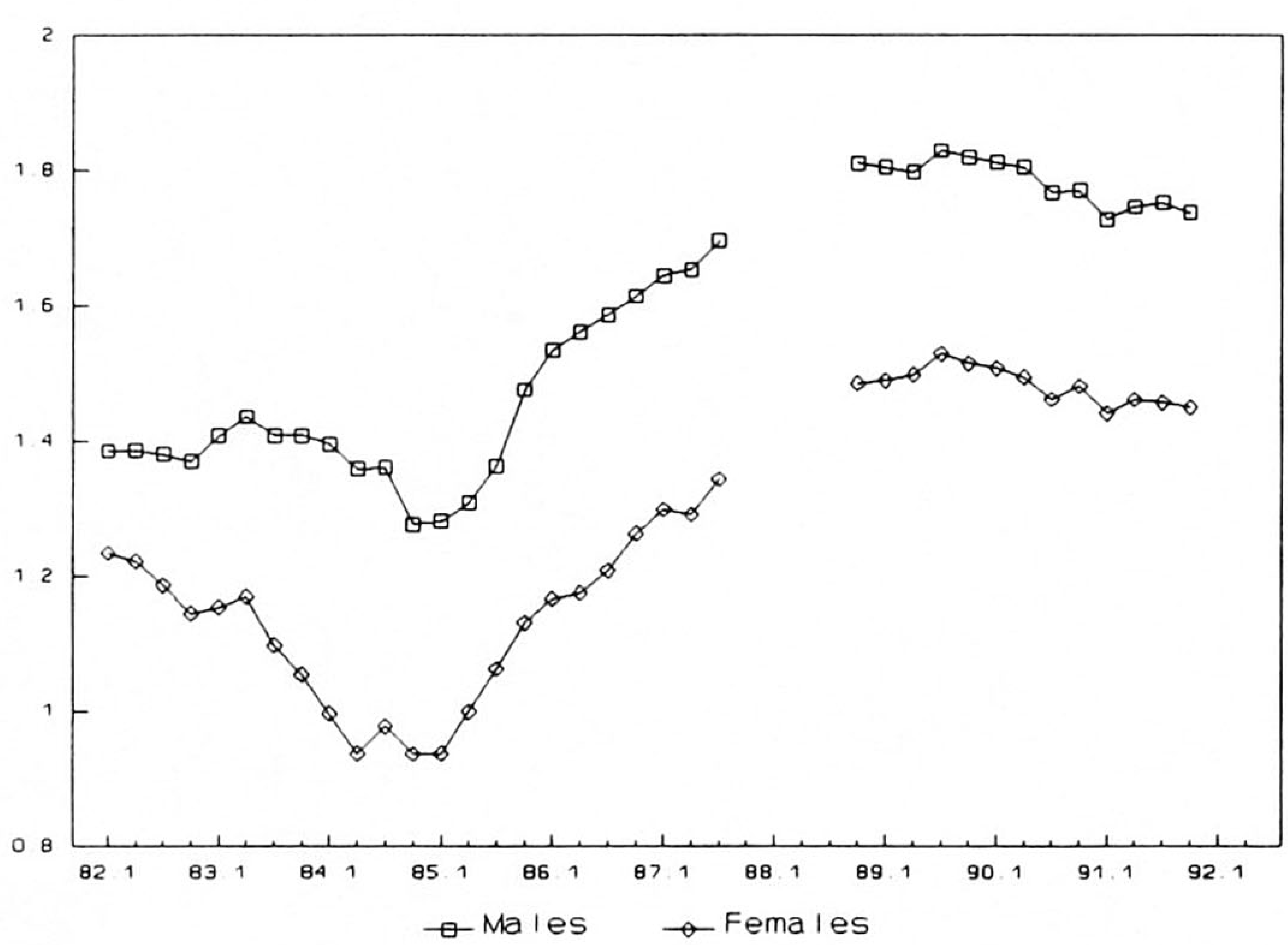
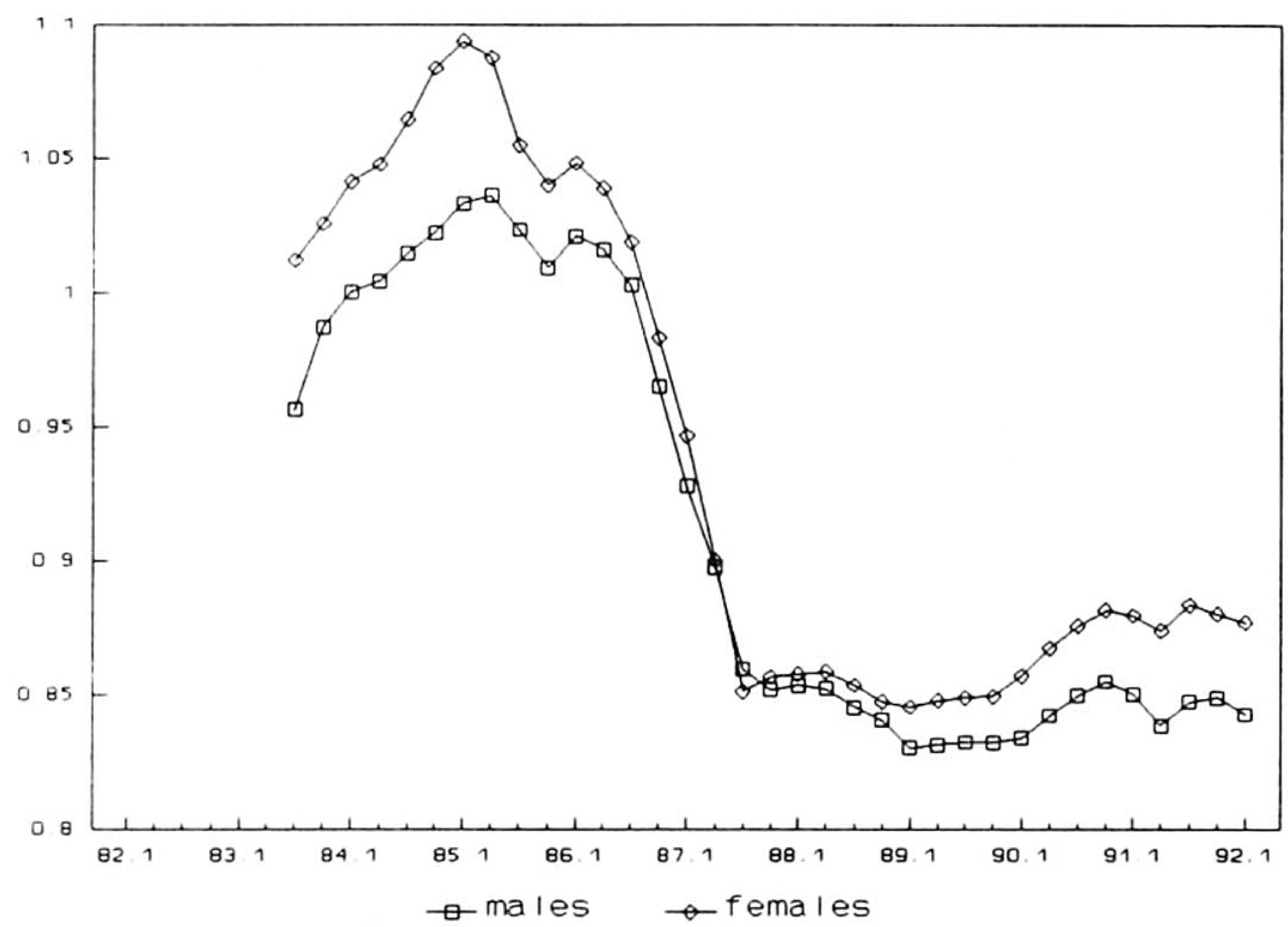


Figure 2. (Continued)

France



UK

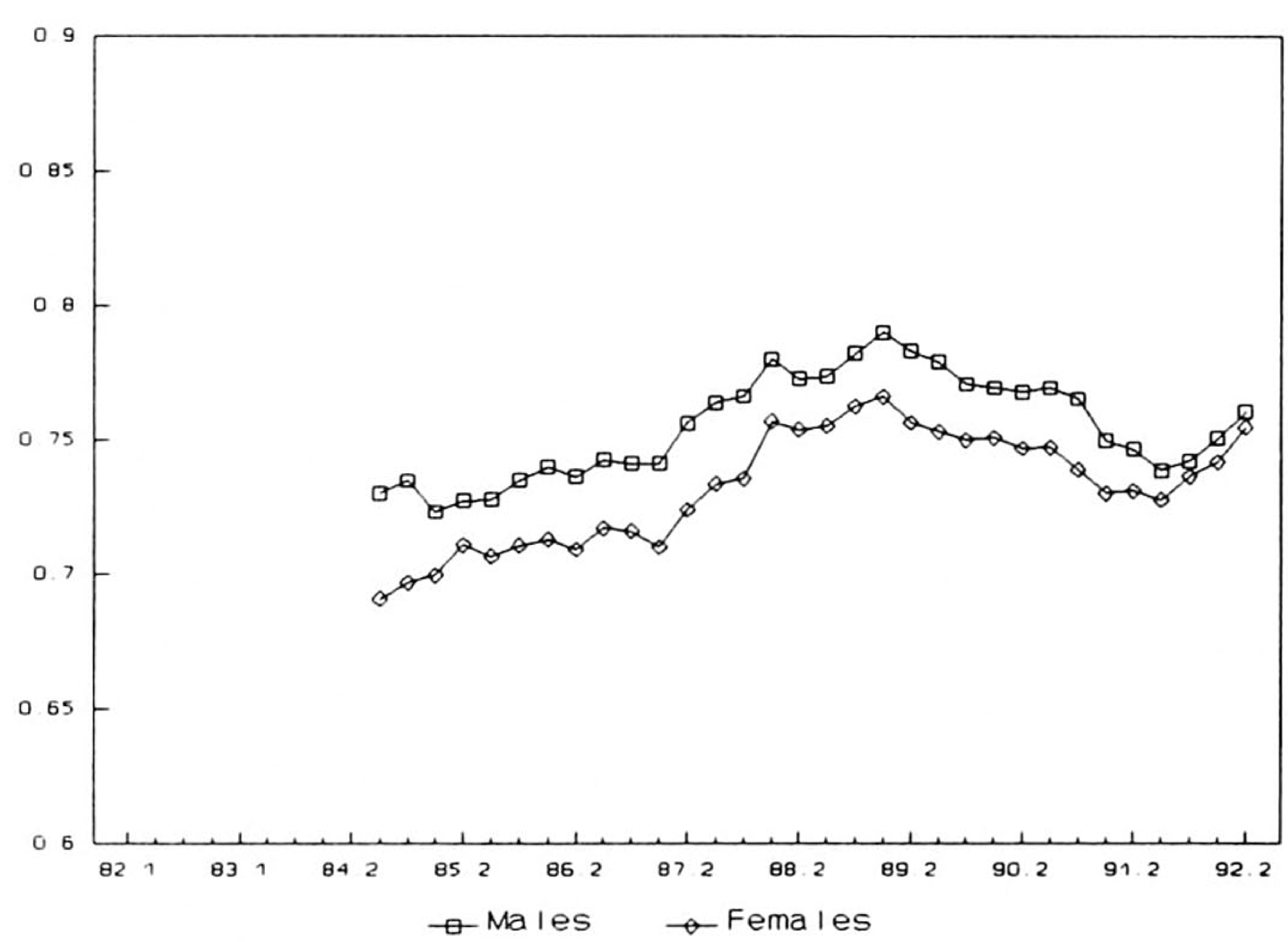


Figure 3. Indicators unobserved heterogeneity; three countries.

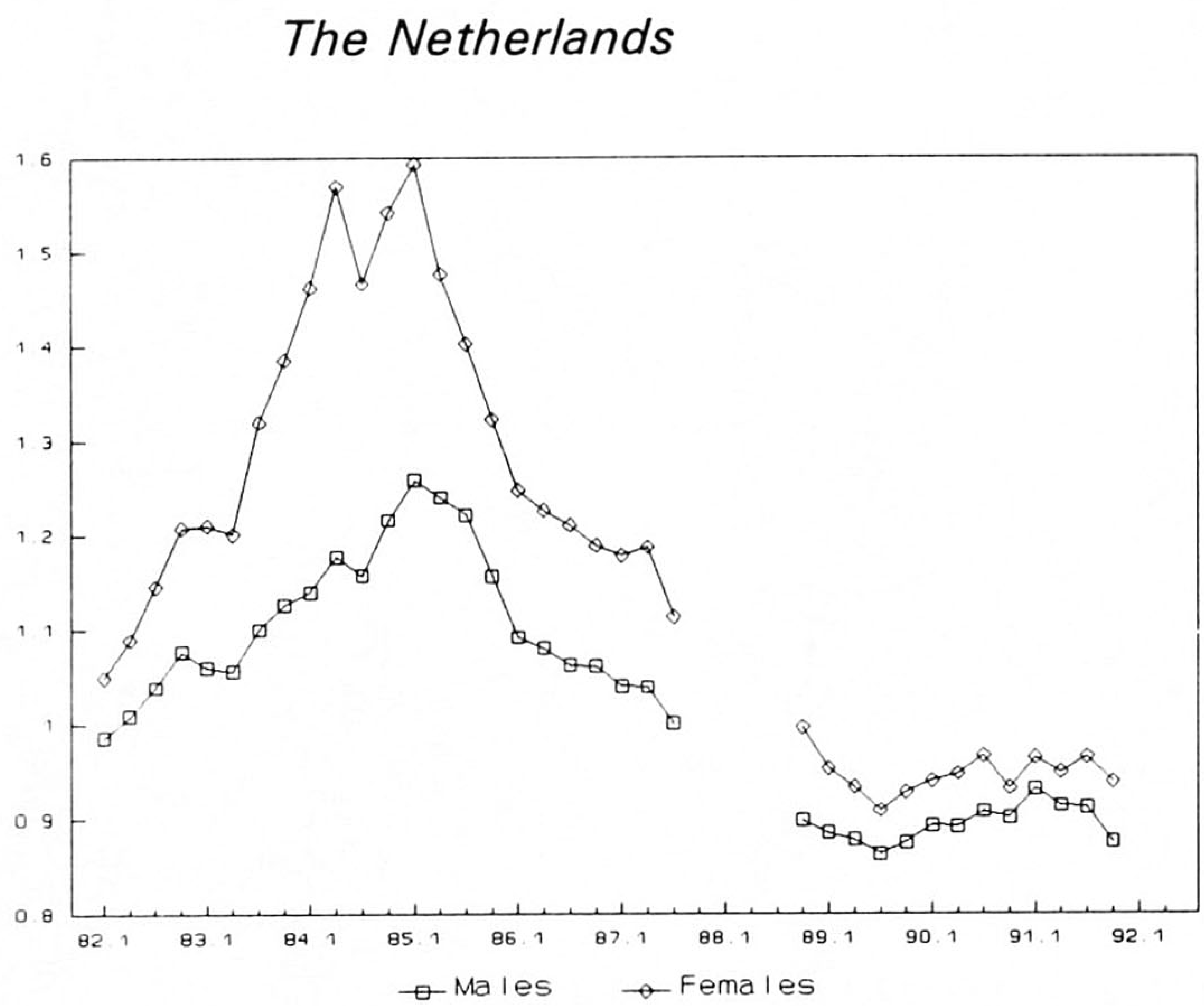


Figure 3. (Continued)

In addition, we merged the graphs for both genders per country. As a result, the length of the range on the vertical axis for The Netherlands exceeds the length for the other two countries. The range on the horizontal axis is the same for each picture.

For British females the difference between the values of the indicator in different steady states is small, while for the other groups the opposite is true. We conclude that state dependence occurs for British males and French and Dutch males and females.

Subsection 4.1 suggests that if state dependence is known to be monotone, then a negative (positive) relationship between the state dependence indicator and the overall exit probability implies that there is negative (positive) state dependence. From Figures 1 and 2 we infer a positive relation for the Dutch and French unemployed, indicating positive state dependence. For the British male unemployed we find a negative relation, indicating negative duration dependence. For British females the state dependence indicator is fairly constant over calendar time, so there is no obvious relation to the level of the overall exit probability. The results are summarized in Table 4.

We have also investigated the sign of the relation more formally by calculating the correlation coefficients between the state dependence indicators and the overall exit probabilities, using data from the steady state periods only. The results are shown in the first two columns of Table 5. They are in accordance to those in Table

Table 4. Steady State Periods and Results of the Eyeball Check on State Dependence

Country	*Steady State With Low Exit Prob.*	*Steady State With High Exit Prob.*	*Eyeball Check on State Dependence Presence*		*Sign (monotone)*	
			Males	*Females*	*Males*	*Females*
France	1984.2–86.4	1987.4–90.4	*yes*	*yes*	+	+
United Kingdom	1984.3–87.1	1989.4–90.4	*yes*	*no*	–	0
Netherlands	1985.2–87.2	1988.4–91.4	*yes*	*yes*	+	+

4. For British females, however, the correlation coefficient is significantly negative. It may be that for this group there is negative state dependence, but that the decrease of the exit probability over the duration of unemployment is small. In a way, this inconclusiveness illustrates the limitations of the eyeball checks. In the absence of a formal statistical framework it is not possible to infer the presence of small but significant effects.

It should be stressed again that the results on the sign of the state dependence are conditional on the assumption of monotone state dependence. There are theoretical as well as empirical indications that under certain circumstances non-monotone state dependence may be important (see Devine & Kiefer, 1991; Van den Berg & Van Ours, 1994 and the references in these).

C. Eyeball Checks on Unobserved Heterogeneity

The eyeball check for unobserved heterogeneity examines the behavior of the heterogeneity indicator over calendar time. This check does not depend on steady state assumptions. From Figure 3 it is obvious that unobserved heterogeneity is important in the French and Dutch unemployment duration data. In the British data, unobserved heterogeneity does not seem to be important. It turns out that for France and The Netherlands the indicators $\theta(t|\tau)/\theta(t-1|\tau)$ with $t \geq 2$ (not reported here) lead to the same conclusion as the indicator graphed in Figure 3. For the United Kingdom, however, the latter indicators suggest the presence of unobserved heterogeneity. This difference in outcome between eyeball checks for the same phenomenon suggests that the model specification may be incorrect for U.K. unemployment durations. This is confirmed below.

As shown in Subsection IV.B, the heterogeneity indicator can also be used for checking the MPH assumption. In particular, a positive relation between this indicator and the—one period lagged—exit probability for the first duration class suggests that the MPH assumption is violated.

We have investigated this by calculating the sample correlation coefficient between the heterogeneity indicator and the lagged exit probability for the first duration class. (For sake of brevity we dot not present the corresponding graphs.)

Table 5. Correlation Coefficients

	Informative on Sign of (monotone) State Dependence[a]		*Informative on Validity of MPH Assumption*[b]	
	Males	*Females*	*Males*	*Females*
France	0.90	0.76	–0.85	–0.83
United Kingdom	–0.98	–0.98	0.75	0.65
Netherlands	0.87	0.95	–0.93	–0.97

Notes: All correlation coefficients differ significantly from zero at 1% level.

a Correlation between the state dependence indicator and the overall exit probability in steady state periods. The sign corresponds to the sign of the monotone state dependence.

b Correlation between the heterogeneity indicator and the lagged exit probability for the first duration class in the total data period. A positive sign corresponds to a misspecified model.

Columns 3 and 4 of Table 5 show the results of these calculations. For France and The Netherlands we find a significant negative relationship, confirming the model specification. For the United Kingdom the correlation is significantly positive. Therefore, for the United Kingdom the MPH assumption is violated. This casts doubt on the results of the other eyeball checks in this chapter for the United Kingdom.

Van den Berg and Van Ours (1994) apply formal econometric tools to estimate unemployment duration models for the same three countries as considered here, using extensive aggregate time series data on exit probabilities from a number of unemployment duration classes. Almost all of their results are in agreement to those presented here. For The Netherlands they find that state dependence is non-monotonic.

VI. CONCLUSION

In this chapter we have examined eyeball checks for state dependence and unobserved heterogeneity in aggregate duration data. We have shown that these checks have excellent properties in terms of the power against the corresponding alternative hypothesis.

The eyeball checks do not rely on parametric assumptions on the determinants of the hazard. This is a marked advantage. It makes the check for state dependence (unobserved heterogeneity) insensitive to the shape of the unobserved heterogeneity distribution (state dependence). The checks are based on the facts that (*i*) if there is no state dependence then the mean duration is inversely proportional to the steady-state impact parameter in the exit probabilities, and (*ii*) if there is no unobserved heterogeneity then the observed exit probabilities are multiplicative in terms of duration and calendar time.

As an empirical illustration, we applied the eyeball checks to aggregate unemployment duration data from three European countries: France, the United Kingdom and The Netherlands. The data are quarterly and distinguish between males and females. The results indicate that in general there is state dependence in unemployment. Under the assumption of monotonicity of state dependence, we find positive state dependence for the French and Dutch unemployed and negative state dependence for British male unemployed. There is no strong indication of state dependence for the British female unemployed. Furthermore, unobserved heterogeneity is important in the French and Dutch unemployment data, while it is not an important phenomenon in the British data.

Finally, our chapter provides eyeball checks of the Mixed Proportional Hazard model framework. For British data this check results in rejection, which casts doubt on the results of the other eyeball checks for the United Kingdom.

APPENDIX A

In this appendix we derive a result that is used in Subsection IV.A.

In the proposition below, IFRA and DFRA stand for Increasing Failure Rate Average and Decreasing Failure Rate Average, respectively (see e.g., Hollander & Proschan 1984; the definitions are stated in the proof of the proposition).

Proposition A1. Consider the continuous-time analog of the model satisfying Assumptions 1, 2, 3′ and 4 in which $\theta(0|s)$ exists and is positive.

(i) The ratio $\theta(0|s)/\overline{\theta}(s)$ does not change with a_s on $<0,\infty>$ if and only if there is no state dependence.

(ii) If $\psi_1(t)$ as a hazard rate is strictly IFRA (for which it is sufficient that $\psi_1(t)$ is strictly increasing) then $\theta(0|s)/\overline{\theta}(s)$ is strictly increasing in a_s.

(iii) If $\psi_1(t)$ as a hazard rate is strictly DFRA (for which it is sufficient that $\psi_1(t)$ is strictly decreasing) then $\theta(0|s)/\overline{\theta}(s)$ is strictly decreasing in a_s.

Proof. In continuous time, $\theta(t|\tau,v),\theta(t|\tau)$ and so forth, are rates rather than probabilities. Equation (4.1) still holds while (4.3) is replaced by

$$\overline{\theta}(s) = 1/\mathrm{E}(t|s) \tag{4.5}$$

(this can be inferred from for example Heckman & Singer, 1984). In order to prove claim (i) we want to characterize when $\overline{\theta}(s)$ is multiplicative in a_s, since $\theta(0|s)$ is always multiplicative in a_s. According to Lancaster (1990), the only continuous-time MPH model for which $\mathrm{E}(t|s)$ is multiplicative in a_s is the model in which the baseline hazard ψ_1 follows a Weibull specification: $\psi_1(t) = \alpha t^{\alpha-1}$, with $0 < \alpha < \infty$. The only α for which $a_s\mathrm{E}(t|s)$ does not depend on a_s is $\alpha = 1$ (which incidentally is the only α for which $0 < \theta(0|s) < \infty$). This means that ψ_1 is constant, so there is no state dependence. In sum, we have proven claim (i).

It can be shown that the continuous-time equivalent of (4.4) (which is the derivative of $a_s E(t|s)$) equals

$$\int_0^{\infty} [t\psi_1(t) - \int_0^t \psi_1(u)du] \cdot E_v \{a_s v \cdot \exp\{-a_s v \cdot \int_0^t \psi_1(u)du\}\}dt$$

If the term in square brackets is positive (negative) for every $t => 0$ then the expression above is positive (negative) for every a_s. In that case $\theta(0|s)/\bar{\theta}(s)$ is strictly increasing (decreasing) in a_s. The term in square brackets is positive for every $t > 0$ if and only if the distribution of $t|s,v$ has strictly IFRA, or, in other words, if $\psi_1(t)$ as a hazard rate has the "strictly IFRA" property. Sufficient for this is that the distribution of $t|s,v$ has strictly IFR (increasing failure rate), or, in other words, that $\psi_1(t)$ is strictly increasing. Similarly, the term in square brackets is negative for every $t > 0$ if and only if the distribution of $t|s,v$ has strictly DFRA. For the latter it suffices that $\psi_1(t)$ is strictly decreasing. □

Somewhat loosely, one might translate IFRA as "a failure rate (here: ψ_1(t)) which is increasing on most time intervals and never strongly decreasing" and DFRA analogously.

APPENDIX B

For $t \in \{2, 3, ..\}$ it is not always true that if $\theta(t|\tau_1)/\theta(t-1|\tau_1) = \theta(t|\tau_2)/\theta(t-1|\tau_2)$ for some τ_1,τ_2 with $\theta(i|\tau_1 - t + i) \neq \theta(i|\tau_2 - t + i)$ for all $i \in \{0, 1, \ldots, t\}$, that then there is no unobserved heterogeneity. Suppose we observe steady states. For a given t, there may be different values of s generating the same value of $\theta(t|s)/\theta(t-1|s)$. For example, if $G(v)$ is discrete with $\Pr(v = 1/5) = 1/2 = 1 - \Pr(v = 3/5)$, then $\theta(2|a_s = 1.15)/\theta(1|a_s = 1.15) = \theta(2|a_s = 1.448)/\theta(1|a_s = 1.448)$. However, if additional values of a_s or t are used in this example then there is no ambiguity anymore.

Indeed, from extensive numerical analyses based on particular $\psi_1(t),\psi_2(\tau)$ and $G(v)$ it follows that if there is unobserved heterogeneity then in most cases $\theta(t|\tau_1)/\theta(t-1|\tau_1) \neq \theta(t|\tau_2)/\theta(t-1|\tau_2)$ when $\theta(i|\tau_1 - t + i) \neq \theta(i|\tau_2 - t + i)$ for all $i \in \{0, 1, \ldots, t\}$. Thus, it seems that in practice one may as well safely use ratios $\theta(t|\tau)/\theta(t-1|\tau)$ with $t \in \{2,3,..\}$ for eyeball checks on unobserved heterogeneity.

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